

Fiber-optic signal processor with applications to matrix-vector multiplication and lattice filtering

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A new fiber-optic signal processor is proposed to implement systolic matrix-vector multipliers and lattice filters. 10^9 multiplications/sec can be achieved with currently available components for matrix-vector multiplications that involve Toeplitz matrices. A 2×2 (Toeplitz) matrix-vector multiplier has been experimentally demonstrated using single-mode fibers and directional couplers. The filtering characteristics of the device are also discussed.

Single-mode-fiber recirculating and tapped delay lines were recently demonstrated^{1,2} with frequency capabilities well above a gigahertz. The extremely high modulation bandwidths (of the order of 100 GHz-km), available because of the low dispersion of the single-mode fiber, strongly suggest further research into more-elaborate fiber structures for implementing additional signal-processing techniques.

In this Letter we report on a novel single-mode fiber-optic scattering processor (FOSP), which consists of two parallel fibers with several couplers that are distributed along the fiber's lengths (see Fig. 1). (The term scattering is used here in the signal-processing context of Ref. 3.) Each coupler operates on the upper-left-hand and lower-right-hand inputs (L_{up} and R_{down}) to produce the upper-right-hand and lower-left-hand outputs (R_{up} and L_{down}). Mathematically, a general (not necessarily passive) linear coupler of this type can be described by (linear) transmission and reflection operators t , τ , r , and ρ according to

$$\begin{bmatrix} R_{up} \\ L_{down} \end{bmatrix} = \begin{bmatrix} t & \rho \\ r & \tau \end{bmatrix} \begin{bmatrix} L_{up} \\ R_{down} \end{bmatrix}. \quad (1)$$

Since the delay between successive couplers can be incorporated easily into the couplers' characteristic matrices, our structure consists of a series of scatterers whose cascaded operation is described by Redheffer's scattering formalism.³ Therefore, with appropriate couplers, the FOSP can simulate (with a fast speed) many physical phenomena that are also described by the same scattering formalism.³

With conventional (passive) directional couplers,⁴ the FOSP can function as an analog matrix-vector multiplier that operates similarly to digital systolic array processors.⁵ (See Refs. 6 and 7 for a different optical systolic processor, which uses light-emitting diodes and a Bragg cell for input and a charge-coupled-device detector for output. The reader who is unfamiliar with systolic arrays and processors is referred to later paragraphs for an explanation of the FOSP that is based on its impulse response.) In a digital systolic system,⁵ data

flow from the computer memory, passing through many processing elements before they return to memory. The interactions of the input data with the flowing partial results permit multiple computations for each input-output memory access. The building block of systolic processors is the inner-product step processor.⁵ In the limit of weak coupling, $rL_{up} + R_{down} \rightarrow L_{down}$ and $L_{up} \rightarrow R_{down}$, so each coupler is an inner-product step processor and the basic time unit⁵ of the FOSP is determined by the loop delay T_i , which should match the switching time of the coupler. Whereas the general systolic architecture can be two dimensional, a FOSP, which uses mechanically or electronically controlled couplers, is one dimensional since only two of the three ports of the basic systolic step processor are optical channels. One of the more important applications of one-dimensional systolic arrays is in matrix-vector multiplication. As described in Ref. 5, the operation of the multiplier involves $2N - 1$ couplers, corresponding to the $2N - 1$ main and off diagonals of the given $N \times N$ matrix. (In the special but important case of a banded matrix,⁵ less than $2N - 1$ will be required.⁶) As the components of the input vector are serially fed into the input line of Fig. 1, the matrix elements enter the couplers progressively in such a way that each coupler sees matrix elements from a single off (or main) diagonal. However, as long as fast switchable couplers are not available, the high inherent bandwidth of our fiber processor can be fully utilized only for matrix-vector multiplications that involve Toeplitz matrices (i.e., matrices in which all the elements along each of the

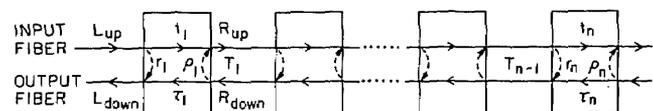


Fig. 1. The scattering processor. The counterpropagating guided waves in the two fibers interact in the couplers. T_i is the round-trip loop delay between the i th and the $(i + 1)$ th couplers.

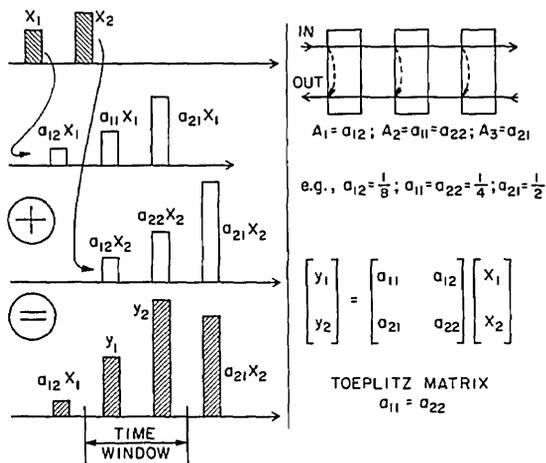


Fig. 2. A systolic Toeplitz matrix-vector multiplier for 2×2 matrices. $\{A_i\}$ are the coupling ratios of the three couplers. The output is the sum of the contributions made by each of the vector components.

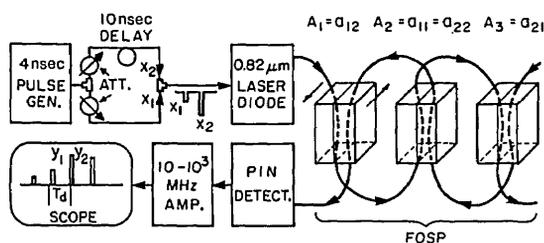


Fig. 3. The experimental device. The coupling ratio A_i of the i th coupler was adjusted by sliding the two half-couplers with respect to each other.⁴ T_d was approximately 10 nsec. Negative electronic pulses were required for positive modulation of the laser light.

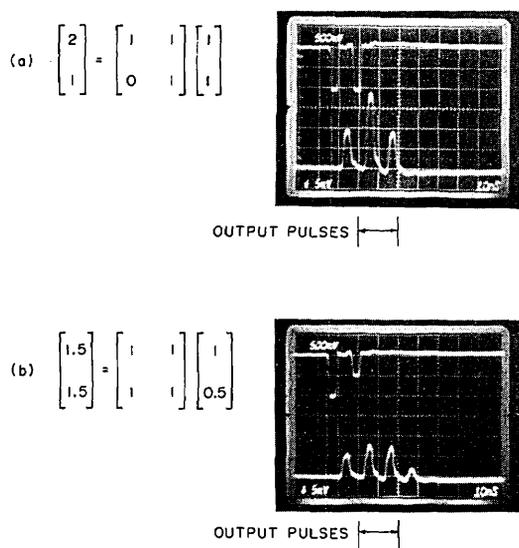


Fig. 4. Experimental matrix-vector products (arbitrary vertical scales). The upper traces show the electronic input pulses. The undershoots that follow the pulses result from the high-pass characteristics of the electronic circuitry. Note that the first and the fourth pulses are proportional to $a_{12}x_1$ and $a_{21}x_2$, respectively.

main and off diagonals are equal). Since convolutions and correlations are described by Toeplitz operators, this last limitation is not too severe.

The operation of the systolic multiplier with fixed weights is described in Fig. 2 for a 2×2 Toeplitz matrix. The matrix elements are represented by the coupling ratios $\{A_i\}$, which are assumed to be weak, thus eliminating recirculation of the input pulses (this restriction is removed later). The components of the input vector enter the input line as a time sequence of light pulses whose spacings are equal to the round-trip loop delay T_d between adjacent couplers. With incoherent light, the device is linear in the intensity of the optical guided waves, and the intensity of the light at the output line is the sum of the responses to the two components of the input vector. Referring to Fig. 2, we see that the matrix-vector product appears in a time slot that follows the first pulse. A similar result holds for the general (Toeplitz) $N \times N$ matrix-vector multiplication. Since such a matrix has $2N - 1$ independent elements, $2N - 1$ couplers will be required, and the N components of the output vector will follow the $(N - 1)$ th output pulse.

In the above discussion, we have neglected recirculations and loop losses. However, as is evident from Fig. 2, proper operation of the multiplier depends only on the device impulse response rather than on the individual coupling ratios and loop losses. Therefore, for a given $N \times N$ (Toeplitz) $[a_{ij}]$ matrix, the $2N - 1$ couplers can be adjusted to yield an impulse response with pulse heights proportional to $a_{1,N}, a_{2,(N-1)}, \dots, a_{i,(N-i+1)}, \dots, a_{N,1}$, respectively. In this case, the weak-coupling requirement can be removed at the expense of a lower data rate since a sequence of delayed pulses resulting from recirculations may follow the output vector and the next input vector must wait for their complete decay.

Although the outlined operation of the device admits of only positive vector components and matrix elements, the extension to the general complex case is straightforward if one uses the previously developed techniques of incoherent optical data processing.⁸ As in the digital systolic case, the main advantage of the FOSP over current transversal matrix-vector multipliers⁹ is its single-output nature, which permits the use of small and therefore fast detectors without sacrificing light-collection efficiency.

An experimental device, shown in Fig. 3, was built from three directional couplers. After the three upper and three lower half-couplers⁴ were spliced separately (by using a fusion splicer), the two sets were assembled to yield three backwardly interconnected couplers. The three couplers were manually adjusted to yield an impulse response with the first three pulse heights proportional to $a_{12}, a_{11} (=a_{22})$, and a_{21} , respectively. The experimental output vectors, which are shown in Figs. 4(a) and 4(b), follow the theoretical predictions quite accurately. Among the factors that determine the ultimate accuracy of the multiplier are incorrect settings of the impulse response, incorrect time delay between the input pulses, unequal loop delays, residual recirculation (for high-data-rate applications), and insufficient frequency bandwidth of the electronic components. (The undershoots that appear after the output

pulses in Fig. 4 are due to the high-pass characteristics of the power splitters and the amplifier.) With smaller components and a closely packed geometry, the delays can be reduced to the order of 0.1 nsec, permitting a 10-GHz data rate. Similar data rates could be obtained if the architecture of Fig. 1 were implemented by using integrated-optics techniques.

When the coupling ratios are small (with respect to 1), the configuration of Fig. 1 acts as a nonrecirculating tapped delay line. Its filtering characteristics have been investigated by using a network analyzer and found to be similar to the results of Refs. 1 and 2. As a tapped delay line, the FOSP has an advantage over the classic tapped delay line² in that it provides internal summation with a subsequent efficient coupling to a small and fast optical detector.

The full scattering nature of the FOSP is revealed only when recirculation is taken into account, i.e., when r and ρ are of the order of unity. Unfortunately, the high splice losses in our experimental device ($\approx 25\%$ /splice) heavily attenuated the recirculating waves. A better device, made of two single strands of fiber with no splices, is currently under construction. With enough recirculation (which can be boosted further by intraloop amplification,¹ our device will serve as a feedback lattice (ladder) filter¹⁰ with an infinite impulse response (as opposed to the finite impulse response of the tapped delay line). This type of filter has certain advantages in terms of the location of poles and zeros and sensitivity to parameter variations and has been used extensively in speech modeling.¹⁰ The dual-feed forward lattice filter¹⁰ can also be implemented simply by replacing the scattering couplers of Fig. 1 by forward couplers, which operate on L_{up} and L_{down} to produce R_{up} and R_{down} . The ability of this last forward-transfer structure to perform matched filtering has already been analyzed and demonstrated using multimode fibers.¹¹

In summary, we have suggested and demonstrated a new fiber-optic device with a lattice structure. Its operation as a systolic matrix-vector multiplier was experimentally verified, as were its filtering characteristics. Future applications have also been suggested.

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