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Study on a Bessel–Gaussian beam laser

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Abstract

A new concept and the method are presented to obtain a laser beam output with high luminance and quality. Instead of using the conventional concept of "obtaining a single transverse mode through compressing the oscillating mode volume using a small aperture diaphragm", the large multimode volume and the high output power are obtained by studying the physical mechanism of the expansion and coupling between a Bessel beam and a Gaussian beam. A high quality light beam (close to the diffraction limit) with high luminance and large intensity difference between the center and the edge is achieved simultaneously.
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Keywords: Bessel–Gaussian beam; Free-space; Waveguide

1. Introduction

Many applications demand a high-quality laser beam that has possibly little diffusion along the transmission path, and whose central intensity should be as high as possible than the edge intensity in the intensity distribution. Generally speaking, the eigenmode Gaussian beam is the best quality beam with the diffraction limit $M^2 = 1$ and the sharply declining intensity distribution with a Gaussian function. In order to obtain an eigenmode operation, people usually put a small aperture diaphragm, a critical angle reflector or other mode-selecting components into the laser tube to compress the oscillating mode volume and repress higher-order modes. However, the laser output power decreases dramatically at the same time due to the compression of the oscillating mode volume.

In this paper, we present a new method which can obtain a laser beam output with high luminance and good quality. And we develop a special resonator (compound resonator) in which a hollow waveguide tube with a proper size is placed between the working materials and the reflective mirror. In the compound resonator, the hollow waveguide tube is a mode-controlling component with no gain effect, which is equivalent to a free-space gain area if the transverse size of the working material is large enough. Based on the principle of the expansion and coupling between a Gaussian beam in the free-space gain area and a Bessel beam that forms in the hollow waveguide tube, the combination of higher-order modes that are expanded by the Bessel beam forms a big oscillating mode volume. The Gaussian modes with different orders participate in the oscillation and amplification. Then the output of a zero-order Bessel beam coupled by the hollow waveguide tube is obtained. By selecting appropriate parameters of the compound resonator, one can obtain a large enough multimode volume in the gain area so as to attain a huge laser power. In the meantime, the laser output with high luminance and quality is also obtained due to the coupling effect of the hollow waveguide tube. This technology is completely different from the conventional method of obtaining an eigenmode beam only by suppressing higher-order modes. In this paper, we carefully analyze the applicability and application prospect of this technology, and prove our prediction by measuring the intensity distribution output of a far-field eigenmode. The central power density of a CO_2 laser and a dye laser which utilize this hollow waveguide tube to control modes is three times higher than that of the original laser beam. By measuring the quality of the output laser beam, we observe an obvious intensity difference between the center and the edge. And the diffusion is close to the diffraction limit.

2. Theoretical analysis

The experimental device is shown as in Fig. 1. The compound resonator is composed of a reflective mirror

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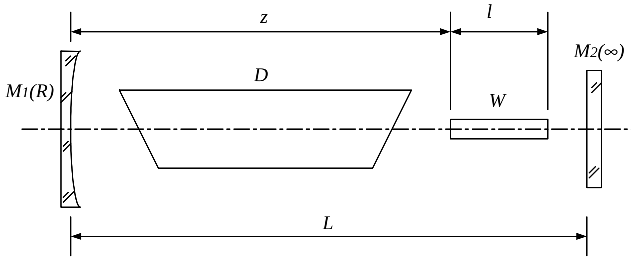

Fig. 1. Setup for "waveguide/free-space" compound cavity: M_1—spherical reflective mirror; M_2—plane output mirror; D-beam-calibre discharge tube; W—hollow waveguide tube.

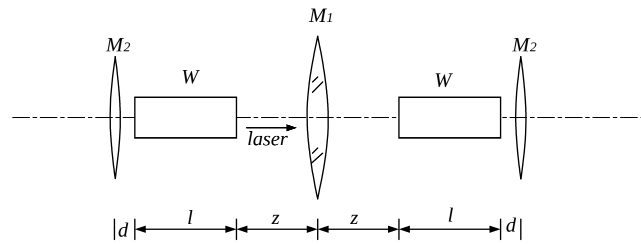

Fig. 2. Equivalent free-space-waveguide-lens system (a): M_1—spherical reflective mirror; M_2—plane output mirror.

M_1, a large aperture discharge tube D, a hollow waveguide tube and an output mirror M_2. The discharge tube D, with a large enough diameter, is full of working gases, that is, equivalent to a free-space gain area of the oscillating mode.

The difference between the compound resonator (shown in Fig. 1) and the common waveguide oscillating resonator [1] lies in the fact that the waveguide tube and the large aperture discharge tube work independently as different components in the compound resonator, whereas in the common waveguide resonator, the waveguide tube is not only a component for the mode-controlling and the beam guidance, but also a container for the gas discharge simultaneously. Within the compound resonator, therefore, the mode controlling effect of the waveguide tube decides the output of the low-order waveguide mode. However, the whole active mode volume of the compound resonator is not confined by the size of a waveguide tube because a waveguide tube has no gain effect.

The oscillating mode that satisfies the self-consistence condition of a compound resonator must be the low-order waveguide eigenmode that may transmit in a waveguide tube with a low loss. At the same time, it must have the "Laguerre–Gaussian" distribution in the free-space gain area so as to be coupled efficiently by the reflective mirror. One of the effective methods to analyze the mode field distribution and the resonator loss is a digital approach [2] using the matrix computing. That is a kind of iterative method that can be used for obtaining the lowest-order oscillating mode with the arbitrary geometrical alignment and its corresponding loss. In other words, the lowest order waveguide mode transmits through the resonator continuously until the reproduced field distribution can be achieved after every transmission.

Fig. 2 is equivalent to the expanded figure of a "free-space-waveguide-lens series" shown in Fig. 1.

Assuming an output mirror M_2 to be a plane mirror and close to the orifice of the waveguide tube, we can ignore the distance between the waveguide tube and the M_2 and then simplify Fig. 2 further to be the equivalent free-space-waveguide-lens system with a half symmetrical structure presented in Fig. 3.

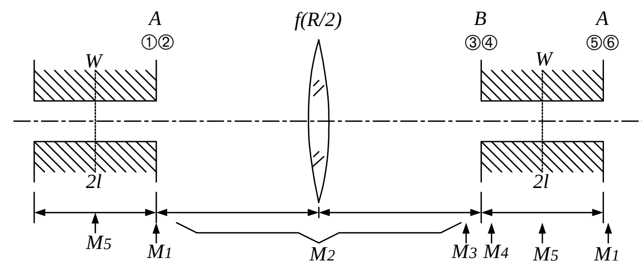

Fig. 3. Equivalent free-space-waveguide-lens system (b): f—equivalent lens of M_1 mirror; W—hollow waveguide tube; M_1–M_5—alternate matrixes.

At the orifice of waveguide tube, the waveguide mode is calculated numerically through a transformation matrix M_1 to the limited grid [3]. This means to expand the waveguide EH_{1m} mode ① to the "Laguerre–Gaussian" TEM mode field ② by calculating the numerical integral of M_1 as follows:

$$(M_1)_{mn} = 2\pi \int_0^a r\psi_m(r)\phi_n(r)\,\mathrm{d}r. \quad (1)$$

In Eq. (1), α is the waveguide tube radius; $\phi_n(r)$ and $\psi_m(r)$ are the normalized functions of a waveguide mode and a free-space mode, respectively. The "Laguerre–Gaussian" mode ② transmits through an equivalent lens f and goes back to the orifice B of the waveguide tube by "Frenel–Kirrchof" diffraction integral. This process can be calculated through a diagonal matrix M_2. On the condition of the reflective mirror radius $R \to \infty, M_2$ can be presented in a much more concise form

$$(M_2)_{mn} = \delta_{mn}\exp[-\mathrm{i}2m\tan^{-1}(4z/k\omega_0^2)], \quad (2)$$

where k is the wave vector and ω_0 is the waist radius of the "Laguerre–Gaussian" light beam. When transmitting to the orifice B, the free-space mode is a TEM mode with the expanded facula $\omega > \omega_0$ and a limited radius R'. This mode must be transformed to a TEM mode with a waist facula ω_0 through the matrix M_3:

$$(M_3)_{mn} = (2/\omega\omega_0)\int_0^\infty \mathrm{d}r^2 L_m(\alpha r^2)L_n(\beta r^2)\exp(-qr^2). \quad (3)$$

In the equation above, $\alpha = 2/\omega_0^2$, $\beta = 2/\omega^2$, $q = 1/\omega_0^2 + 1/\omega^2 - \mathrm{i}k/2R$,

$$\omega = \omega_0[1 + (4z/k\omega_0^2)^2]^{1/2}, \quad R = 2z[1 + (k\omega_0^2/4z)^2],$$

where z is the distance between the orifice B and a reflective mirror; L_m and L_n are Laguerre polynomials of the order m and n, respectively. M_4, the inverse matrix of M_1, then transforms this free-space mode to the waveguide mode once again.

Shown as the following, M_5 is a transmission matrix of the waveguide mode in the waveguide tube:

$$(M_5)_{mn} = \delta_{mn}\exp[-u_{1m}^2(1/ka^2)(\mathrm{Re}\, v_n/k_a)]$$
$$\exp\left[-\frac{\mathrm{i}}{2}(u_{1m}^2 - u_{11}^2)(l/ka^2)(1 + 2I_m v_n/k_a)\right]. \quad (4)$$

Beginning from A (shown in Fig. 1), the oscillating mode field achieves the reproduction of the original field distribution in the place A' after a round of reciprocation. The final round-trip matrix is

$$M = M_5 M_4 M_3 M_2 M_1, \quad (5)$$

$$MX_i = \Lambda_i X_i. \quad (6)$$

M can be worked out from Eq. (6). X_i, the eigenvector of M, represents a field distribution of the oscillating mode in the compound resonator. Λ_i is the eigenvalue of M and the total loss of the compound resonator is attained from the following equation:

$$L_i = 1 - |\Lambda_i|^2. \quad (7)$$

The results of the numerical calculation show that the mode field distribution and the resonator loss of the compound resonator depend on the parameters of the resonator structure, including the length of the waveguide tube l, the tube radius α, the reflective mirror radius R, the distance z and the parameter, ω_0/α. Actually, ω_0/α is the most important parameter among them. Although the matrix eigenvalue Λi and ω_0/α are independent of each other and the value of ω_0/α seems arbitrary, the number of TEM modes that can be expanded is finite in fact. The sub-matrices M_1, M_2, M_3 show that the value of ω_0/α is related to the active mode volume in the free-space gain area and the convergence of the free-space mode when it returns to the waveguide.

Abrams [4] has even assumed that there only allowed the existence and transmission of the EH_{11} mode, i.e. the waveguide mode with the lowest order, in the waveguide tube. Consequently, if the reflective mirror is placed properly, the 98% energy of EH_{11} mode radiates into TEM_{00} mode with a waist facula $\omega_0 = 0.6435\alpha$. This method is probably the most convenient for lasers with waveguide resonator. For the "waveguide-free space" compound resonator, however, the most interesting parts are the entire volume of the expanded TEM mode in the free-space gain area and the total loss in the process that the TEM mode is re-coupled to the waveguide mode again. By the numerical calculation, the waveguide loss of the EH_{1m} modes in the resonator is very close to the loss of the EH_{11} mode. The much more practical concern therefore is that the resonator mode should be the linear combination of the EH_{1m} modes. This means that an appropriate value of ω_0/α should make the EH_{1m} mode coupled into the free-space TEM_{0m} mode at the optimal efficiency.

Based on the expanded equation of the entire energy Q of EH_{1m} modes, we can obtain

$$Q = \sum_{p=0}^{\infty} |A_p(\omega_0)|^2 = |A_0(\omega_0) + A_1(\omega_0) + A_2(\omega_0) + \cdots|^2. \quad (8)$$

In the above equation, $A_p(\omega_0)$ is the amplitude of the p-th free-space expanded mode.

Based on Abram's assumption, we can also obtain

$$\frac{\partial A_0(\omega_0)}{\partial \omega_0} = 0.$$

This is to say that when ω_0/α=0.6435, the 98% energy of the EH_{11} mode is coupled into the TEM_{00} mode, and the rest of 2% energy radiates into the TEM_{0m} mode. Concerning that the resonator mode is the linear combination of low-order EH_{1m} modes (for instance: $EH_{11}, EH_{12}, EH_{13}$), however, we then select the another value of ω_0/α in order that the largest value of $A_p(\omega_0)$ of Eq. (8) appears in the latter terms of this equation. For the term $A_2(\omega_0)$ as an example, we obtain $\omega_0/\alpha = 0.490$ by calculating the equation $\partial A_2(\omega_0)/\partial\omega_0 = 0$. The energy distribution in this way becomes: 89% energy of the EH_{11} mode is coupled into TEM_{00}mode; 84% energy of the EH_{12} mode is coupled into the TEM_{01}mode; 73% energy of the EH_{13} mode is coupled into the TEM_{02} mode. Obviously, the selection of $\omega_0/\alpha = 0.490$, compared with $\omega_0/\alpha = 0.6435$, is able to guarantee much larger oscillating mode volume in the free-space gain area.

According to the calculation, the energy proportion of each waveguide mode in the linear combination of EH_{1m} modes declines promptly with the enhancement of the number of the modes: m. The first three modes almost contain the entire energy of the resonator modes, and especially EH_{11} mode covers the highest proportion. Hence, from experiments we observe the output of the quasi-Gaussian ground mode with the far-field distribution both at the two values of ω_0/α mentioned above.

The characteristic of the "waveguide-free-space compound resonator" is that the EH_{1m} mode confined by the waveguide tube can be expanded to free-space modes of different orders at the orifice of the waveguide tube. Those modes are amplified and then form a large active mode volume in the process of the transmission in the free-space gain area. When the re-coupled waveguide modes feed back to the orifice of the waveguide tube, its energy must be larger than the energy produced in the oscillation of single Gaussian ground mode. The processes of expansion, amplification and coupling between free-space modes and the waveguide modes produce a new mode-selecting concept of "multimode oscillation and single mode output". Hence, we provide a method to achieve a laser output with high power and single mode operation.

3. Experimental methods and results

3.1. The parameters of the compound resonator

The selection for the radius a and the length l of a waveguide tube in the compound resonator should satisfy the matrix M_5. This means, these parameters should make the lowest order waveguide mode transmit with a small enough loss in the tube, and simultaneously avoid the oscillation of higher-order modes. Generally, the Fresnel number N should be selected from $N = \alpha/L\lambda = 0.2 \sim 1$. L is the length of the compound resonator and λ is the wavelength. The tube length l may have an impact on the proportion of the energy, the transmission loss and the number of low-order modes that the waveguide tube can possibly form. We find that the impact is not so obvious if the ratio l/α is over 20 in our primary experiments.

The three parameters, the reflective mirror radius R and distance z and ω_0/α, decide the oscillating mode volume and the coupling loss of the reflective mirror. In the early stage of study, we still adopt Abram's results [5] and think that the parameters of the low coupling loss area: $z = f$ and $R = 2f(f = \pi\omega^2/\lambda$; f: confocal parameter.), may be one of the applicable configurations for the compound resonator (There maybe exists better configuration.). In order to compare the influence of the values of ω_0/α to the oscillating mode volume, according to the calculative results, two values of ω_0/α are selected as: (1) $\omega_0/\alpha = 0.6435$, corresponding to $\partial A_0(\omega_0)/\partial\omega_0 = 0$; (2) $\omega_0/\alpha = 0.55$, corresponding to $\partial A_1(\omega_0)/\partial\omega_0 = 0$.

For the selection of the diameter D_T of the big aperture discharge tube which is placed between the reflective mirror and the orifice of waveguide tube, one should consider the largest oscillating mode volume that can exist and accommodate possibly, the diffusion cooling effect of the working materials and the transverse distribution of the inverted population density in the discharge tube. Research [6] shows that it is an appropriate selection when D_T and the diameter D_i of the oscillating light beam satisfy the relationship $D_T = e^2 D_i$.

3.2. Experimental results

In the experiment, the diameter of the discharge tube is $D_T = 28$ mm, the length is $L = 90$ cm, the mixed ratio of the working gases is $CO_2 : N_2 : He : Xe = 1 : 1.25 : 5 : 0.5$, the total pressure is 20 Torr, the discharge current is 35 mA, the waveguide tube radius α is 3.5 mm; the tube lengths l are 10, 50, 100, 200, 400, 1200 mm, respectively.

The experimental setup for measuring the beam profiles (shown in Fig. 4) is composed of a small aperture diaphragm, a power meter and an oscillagraph. The laser beam emits towards the power meter horizontally and the small aperture diaphragm makes a vertical motion up and down in front of the power meter (the motion distance is over the beam diameter). The signals out of the power meter are input to an oscillagraph when that small aperture diaphragm finishes a scanning. The beam profiles are observed and then printed out from the oscillagraph. After removing the small aperture diaphragm, we obtain the total output power. And the average power density is also obtained through the equation of $D_{average} = P_{total}/D_{beam}$($D_{average}$: average power density; P_{total}: total output power; D_{beam}: beam diameter)

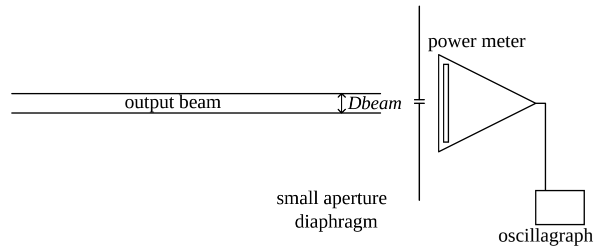

Fig. 4. The setup for measuring the output power and beam profiles.

Experimental results show that when there is no waveguide tube in the resonator, i.e. only common CO_2 laser, the output pattern of the typical multimode is observed and the average power density is 21 W/cm^2 on the conditions of the reflective mirror radius $R = 3$ m, the output mirror radius $R = \infty$ and the total length of the resonator $L = 1.8$ m.

When a diaphragm with radius $a_T = 6$ mm (equivalent to $N = 1.5$) is placed in the resonator, the output pattern of the quasi-TEM_{00} mode is observed and the average power density is $36W/cm^2$.

The diaphragm in the resonator is then replaced by a waveguide tube with radius $\alpha = 3.5$ mm. At the value of $\omega_0/\alpha = 0.6435$ and based on the low loss condition of $z = f$, $R = 2f$, the distance z between the waveguide orifice and the reflective mirror is fixed at $z = f = 1.5$ m and the mirror M_1 radius R is 3 m long.

For the short waveguide tube ϕ 7×10 mm, it is impossible to form the effect of the waveguide mode just as our expectation. Instead, it becomes a small Fresnel number resonator virtually [7]. The typical ring ground mode pattern (Fig. 5) with the circle symmetrical distribution, similar to the mode pattern output of the confocal unstable resonator [8], can be observed. Since there is a very large diffraction loss in the small aperture resonator, the laser power reduces obviously and the average output power density is only 10 W/cm^2.

For a waveguide tube ϕ 7×50 mm as a replacement, we observe the mode competition that the homogeneously distributed waveguide modes and the ring ground mode of a small aperture resonator appear alternately. The output power density of the waveguide modes is about 30 W/cm^2. Besides, observation shows that the luminance of the waveguide modes is much higher than that of the ring modes.

When placing a waveguide tube ϕ 7×100 mm in the resonator, we obtain a stable output of the single mode and observe the obvious enhancement of the facula luminance. The output power density is 60 W/cm^2, twice as much as that of

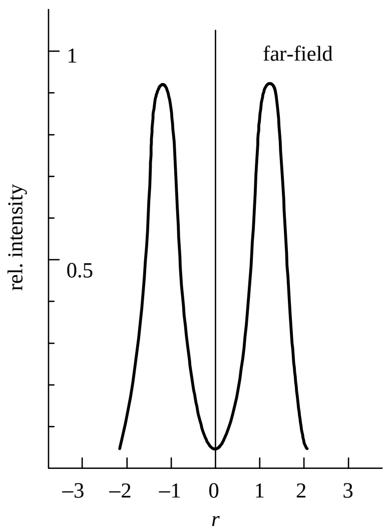

Fig. 5. Output beam intensity and facula of small-N cavity.

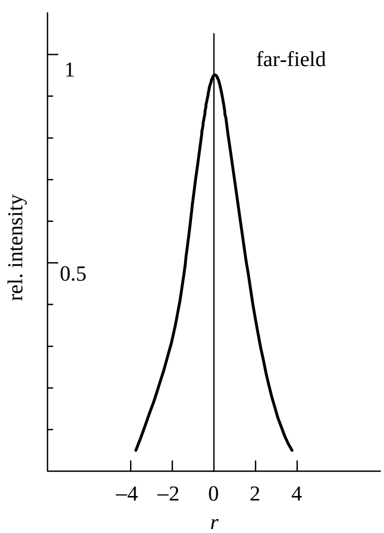

Fig. 6. Output beam intensity and facula of "waveguide free-space" cavity ($\omega_0/\alpha = 0.6435, f = 1.5$ m, $R = 2f = 3$ m, $z = f = 1.5$ m).

the common resonator. Fig. 6 is far-field intensity distribution of the output beam, with the pattern of the quasi-TEM_{00} mode.

If replaced with a longer waveguide tube, the laser has the same stable single mode output. In the experiment, a waveguide tube $\phi\ 7 \times 100$ mm is purposely put into the resonator and the total length of the resonator $L = 3100$ mm. When the mirror M_1 radius $R = 3$ m, a common plane-concave resonator became an unstable resonator [9], whereas the compound resonator can still keep a stable single mode output, which confirms the operation mechanism of the "compound resonator" as well. Table 1 shows the output mode patterns and the average output power density with different lengths of the waveguide tubes.

Table 1
Output mode patterns and the average output power density with different lengths of the waveguide tubes ($\omega_0/\alpha = 0.6435; a = 3.5$ mm; $z = f = 1.5$ m; $R = 3$ m)

Length of the waveguide tube (mm)	Output mode pattern	Average output power density (W/cm^2)
10	Ring ground mode pattern with the circle symmetrical distribution	10
50	Homogeneously distributed waveguide modes and the ring ground mode (appearing alternately)	30 (the waveguide modes)
100	Stable single mode	60

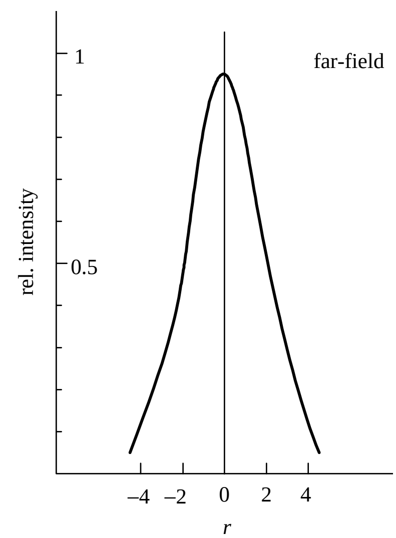

Fig. 7. Output intensity and facula compound cavity ($\omega_0/\alpha = 0.55, f = 1.1$ m, $R = 2f = 2.2$ m, $z = f = 1.1$ m).

For comparison, we use a waveguide tube with a same size, $\phi\ 7 \times 100$ mm, to test the beam output on the conditions of the second value of $\omega_0/\alpha = 0.55$, the corresponding confocal parameter $f = 1.1$ m, the distance $z = f = 1.1$ m and the reflective mirror radius $R = 2.2$ m. If other parameters are invariable, we observe that the laser output power is obviously higher than that of the first value of $\omega_0/\alpha = 0.6435$. Both the similar quasi-ground mode with a far-field intensity distribution and a homogeneous facula pattern are still achieved (shown in Fig. 7). We also measure the oscillating mode volume in the free-space gain area at two different values of ω_0/α, respectively, and observe that the facula size between the mirror M_1 and the discharge tube D is larger than the expected size of TEM_{00} mode. The facula size at a value of $\omega_0/\alpha = 0.55$ is much larger than the one at a value of $\omega_0/\alpha = 0.6435$, which confirms the theoretical analysis.

M^2, i.e. times-diffraction-limit-factor [10], can be defined as the following:

$$M^2 = \frac{\pi}{4\lambda} d_\sigma \cdot \theta_\sigma. \quad (9)$$

In the above equation, d_σ is the diameter of the beam waist, θ_σ is the divergence angle of the far-field beam. The M^2 of the output beam of the compound resonator at a value of $\omega_0/\alpha = 0.55$ is measured experimentally, and the result is $M^2 = 1.106$. But the M^2 of the output beam of the common resonator is $M^2 = 1.5$.

4. Conclusion

The essence of the principle and the method proposed in this paper is the multimode oscillation and a single mode output. This means that the lowest order mode output is obtained by coupling free-space modes of different orders which participate in the oscillation, not just suppressing higher-order modes as the loss. The output beam quality and the power density are therefore both raised greatly. Our study has indicated that the output characteristics of the resonator are not only related to the parameter ω_0/α, but also close to other structural parameters. For examples, a higher oscillating mode volume and the output power have been achieved if adopted the "waveguide/free-space" mixed unstable resonator structure [11,12]. The most appropriate resonator structure can be attained through further theoretical and experimental research. Surely speaking, this novel technology can be applied to the solid-state, gas and liquid lasers other types of lasers, etc. Especially for the high power lasers which are not easy to output high quality beams, the transverse-flow CO_2, CO lasers, chemistry lasers for examples, it has an extensive application prospect.

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